Pensieve header: The Objects. Continues pensieve://Projects/SL2Portfoilo2/Objects.nb.

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The Objects

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“Define” Code

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```
SetAttributes[Define, HoldAll];
Define[def_, defs__] := (Define[def]; Define[defs];);
Define[op_is__ = ℰ_] := Module[{SD, ii, jj, kk, isp, nis, nisp, sis}, Block[{i, j, k},
   ReleaseHold[Hold[
      SD[op_{nisp,$k_Integer}, PP_Boot@Block[{i, j, k}, op_{isp,$k} = ℰ; op_{nis,$k}]];
      SD[op_isp, op_{{is},$k}]; SD[op_sis__, op_{sis}];
     ] /. {SD → SetDelayed,
      isp → {is} /. {i → i_, j → j_, k → k_},
      nis → {is} /. {i → ii, j → jj, k → kk},
      nisp → {is} /. {i → ii_, j → jj_, k → kk_}
     }] ]]
```

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Symmetric Algebra Objects

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```
sm_{i_,j_→k_} := Λ2𝔼_{{i,j}→{k}}[b_k (β_i + β_j) + t_k (τ_i + τ_j) + a_k (α_i + α_j) + y_k (η_i + η_j) + x_k (ξ_i + ξ_j)];
sΔ_{i_→j_,k_} := Λ2𝔼_{{i}→{j,k}}[β_i (b_j + b_k) + τ_i (t_j + t_k) + α_i (a_j + a_k) + η_i (y_j + y_k) + ξ_i (x_j + x_k)];
sS_{i_} := Λ2𝔼_{{i}→{i}}[-β_i b_i - τ_i t_i - α_i a_i - η_i y_i - ξ_i x_i];
sη_{i_} := Λ2𝔼_{{}→{i}}[0];
sϵ_{i_} := Λ2𝔼_{{i}→{}}[0];
```

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```
sσ_{i_→j_} := Λ2𝔼_{{i}→{j}}[β_i b_j + τ_i t_j + α_i a_j + η_i y_j + ξ_i x_j];
sϒ_{i_→j_,k_,l_,m_} := Λ2𝔼_{{i}→{j,k,l,m}}[β_i b_k + τ_i t_k + α_i a_l + η_i y_j + ξ_i x_m];
```

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The CU Definitions

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$$\mathrm{c}\Lambda = \left(\eta_i + \frac{e^{-\alpha_i-\epsilon\beta_i}\,\eta_j}{1+\epsilon\,\eta_j\,\xi_i}\right) y_k + \left(\beta_i + \beta_j + \frac{\mathrm{Log}[1+\epsilon\,\eta_j\,\xi_i]}{\epsilon}\right) b_k + \left(\alpha_i+\alpha_j+\mathrm{Log}[1+\epsilon\,\eta_j\,\xi_i]\right) a_k + \left(\frac{e^{-\alpha_j-\epsilon\beta_j}\,\xi_i}{1+\epsilon\,\eta_j\,\xi_i} + \xi_j\right) x_k;$$

$$\mathrm{Define}\Big[\mathrm{cm}_{i,j\to k} = \Lambda 2\mathbb{E}_{\{i,j\}\to\{k\}}\Big[\left(\eta_i + \frac{e^{-\alpha_i-\epsilon\beta_i}\,\eta_j}{1+\epsilon\,\eta_j\,\xi_i}\right) y_k + \left(\beta_i + \beta_j + \frac{\mathrm{Log}[1+\epsilon\,\eta_j\,\xi_i]}{\epsilon}\right) b_k + \left(\alpha_i+\alpha_j+\mathrm{Log}[1+\epsilon\,\eta_j\,\xi_i]\right) a_k + \left(\frac{e^{-\alpha_j-\epsilon\beta_j}\,\xi_i}{1+\epsilon\,\eta_j\,\xi_i} + \xi_j\right) x_k\Big]\Big]$$

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$$\mathrm{Define}\big[\mathrm{c}\sigma_{i\to j} = \mathrm{s}\sigma_{i,j} \;/.\; \tau_i \to 0,\ \mathrm{c}\epsilon_i = \mathrm{s}\epsilon_i,\ \mathrm{c}\eta_i = \mathrm{s}\eta_i,\ \mathrm{c}\Delta_{i\to j,k} = \mathrm{s}\Delta_{i\to j,k},\ \mathrm{cS}_i = \mathrm{sS}_i \;//\; \mathrm{s}\Upsilon_{i\to 1,2,3,4} \;//\; \mathrm{cm}_{4,3\to i} \;//\; \mathrm{cm}_{i,2\to i} \;//\; \mathrm{cm}_{i,1\to i}\big];$$

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Booting Up QU

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$$\mathrm{Define}\big[\mathrm{a}\sigma_{i\to j} = \Lambda 2\mathbb{E}_{\{i\}\to\{j\}}[a_j\,\alpha_i + x_j\,\xi_i],\ \mathrm{b}\sigma_{i\to j} = \Lambda 2\mathbb{E}_{\{i\}\to\{j\}}[b_j\,\beta_i + y_j\,\eta_i]\big]$$

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$$\mathrm{Define}\big[\mathrm{am}_{i,j\to k} = \Lambda 2\mathbb{E}_{\{i,j\}\to\{k\}}\big[(\alpha_i+\alpha_j)\,a_k + (\mathcal{A}_j^{-1}\,\xi_i + \xi_j)\,x_k\big],\ \mathrm{bm}_{i,j\to k} = \Lambda 2\mathbb{E}_{\{i,j\}\to\{k\}}\big[(\beta_i+\beta_j)\,b_k + (\eta_i + e^{-\epsilon\beta_i}\,\eta_j)\,y_k\big]\big]$$

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$$\mathrm{Define}\Big[R_{i,j} = \mathrm{Module}\Big[\{k\},\ \Lambda 2\mathbb{E}_{\{\}\to\{i,j\}}\Big[\hbar\,a_j\,b_i + \sum_{k=1}^{\$k+1} \frac{(1-e^{\epsilon\hbar})^k\,(\hbar\,y_i\,x_j)^k}{k\,(1-e^{k\epsilon\hbar})}\Big]\Big]\Big]$$

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Three types of inverses appear below!
$\overline{R}$ is the inverse of R in the algebra $\mathbb{B}\otimes\mathbb{A}$.
P is the inverse of R as a quadratic form, like how an element of $V^*\otimes V^*$ can be the inverse of an element of $V\otimes V$.
$\overline{\mathrm{aS}}$ is the inverse of aS as an operator form, like how an element of $V^*\otimes V$ can be the inverse of another element of $V^*\otimes V$.

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In[ ]:=

```
Define[R̄i,j = If[$k == 0, 𝔼{}→{i,j}[-ℏ aj bi - ℏ xj yi / Bi],
    Append[R̄{i,j},$k-1, -Last[PadRight[R̄{i,j},0, $k + 1] R1,2 PadRight[R̄{3,4},$k-1, $k + 1] //
        (bmi,1→i amj,2→j) // (bmi,3→i amj,4→j)]]]
]
```

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```
Define[Pi,j = If[$k == 0, 𝔼{i,j}→{}[βj αi / ℏ + ηj ξi / ℏ], Append[P{i,j},$k-1,
    -Last[R1,2 // (PadRight[P{i,1},0, $k + 1] * PadRight[P{2,j},$k-1, $k + 1])]]]]
Define[P̄i,j = If[$k == 0, 𝔼{i,j}→{}[-βj αi / ℏ + -ηj ξi 𝒜i / ℏ], Append[P̄{i,j},$k-1,
    -Last[R̄1,2 // (PadRight[P̄{i,1},0, $k + 1] * PadRight[P̄{2,j},$k-1, $k + 1])]]]]
```

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```
Define[aSi = (aσi→2 R̄1,i) // P2,1,
  aS̄i = (aσi→2 R1,i) // P̄2,1,
  bSi = (bσi→1 R̄i,2) // P2,1,
  bS̄i = (bσi→1 Ri,2) // P̄2,1
]
```

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Booting Up QU

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```
Define[
  aΔi→k,j = (R1,j R2,k) // bm1,2→3 // Pi,3,
  bΔi→k,j = (Rj,1 Rk,2) // am1,2→3 // P3,i]
```

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```
Define[
  dmi,j→k = ((sϒi→4,4,1,1 // aΔ1→2,1 // aΔ2→3,2 // aS̄3) (sϒj→-1,-1,-4,-4 // bΔ-1→-2,-1 // bΔ-2→-3,-2)) //
    (P3,-1 P1,-3 am2,-4→k bm4,-2→k)]
```

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```
Define[dσi→j = aσi→j bσi→j,
  dϵi = sϵi, dηi = sηi,
  dSi = sϒi→1,1,2,2 // (bS1 aS̄2) // dm2,1→i,
  dS̄i = sϒi→1,1,2,2 // (bS̄1 aS2) // dm2,1→i,
  dΔi→j,k = (bΔi→3,1 aΔi→2,4) // (dm3,4→k dm1,2→j)]
```

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```
Define[C_i = Λ2𝔼_{{}→{i}}[-ħ/2 (b_i + ϵ a_i)],
  C̄_i = Λ2𝔼_{{}→{i}}[ħ/2 (b_i + ϵ a_i)],
  Kink_i = (R_{1,3} C̄_2) // dm_{1,2→1} // dm_{1,3→i},
  K̄ink_i = (R̄_{1,3} C_2) // dm_{1,2→1} // dm_{1,3→i}]
```

Not yet verified

Note. $t == \epsilon a - b$ and $b == -t + \epsilon a$.

```
Define[b2t_i = Λ2𝔼_{{i}→{i}}[α_i a_i + β_i (ϵ a_i - t_i) + ξ_i x_i + η_i y_i],
  t2b_i = Λ2𝔼_{{i}→{i}}[α_i a_i + τ_i (ϵ a_i - b_i) + ξ_i x_i + η_i y_i]]
```

The Knot Tensors

```
Define[kR_{i,j} = (R_{i,j} // (b2t_i b2t_j)) /. t_{i|j} → t,
  k̄R_{i,j} = (R̄_{i,j} // (b2t_i b2t_j)) /. {t_{i|j} → t, T_{i|j} → T},
  km_{i,j→k} = ((t2b_i t2b_j) // dm_{i,j→k} // b2t_k) /. {t_k → t, T_k → T, τ_{i|j} → 0},
  kC_i = (C_i // b2t_i) /. t_i → t,
  k̄C_i = (C̄_i // b2t_i) /. t_i → t,
  kKink_i = (Kink_i // b2t_i) /. {t_i → t, T_i → T},
  k̄Kink_i = (K̄ink_i // b2t_i) /. {t_i → t, T_i → T}]
```